Int. J. Heat Mass Transfer. Vol. 28, No. 2, pp. 331–337, 1985
Printed in Great Britain

0017-9310/85 $3.00 + 0.00
Pergamon Press Ltd.

Freeze-off conditions of a pipe containing a flow of water

T. HIRATA
Department of Mechanical Engineering, Shinshu University, Nagano 380, Japan

and

M. ISHIHARA
Toyota Motor Ltd., Toyota 471, Japan

(Received 14 *March* 1984 *and in revised form* 9 *April* 1984)

Abstract—The ice-band structure of which steady-state ice profile had a variation of flow passage with a cyclic pattern of contractions and expansions along the length of a pipe was investigated experimentally. Correlation equations were proposed for the spacing of the ice-bands, the heat transfer coefficient in the contraction region and the friction factor in the pipe. The conditions for pipe freeze-off were obtained by introducing a modified Reynolds number based on a total pressure.

INTRODUCTION

FREEZING of flowing water in a pipe causes many undesired consequences, such as pressure drop, diminution of flow rate and, sometimes, breakage of a pipe as a result of flow blockage by ice. A number of theoretical and experimental studies have been reported for the problem of pipe freezing; for example, the effects of solidification on heat transfer and pressure drop in a steady state [1, 2], and the profile of solid–liquid interface in a transient freezing process [3, 4] have been investigated for laminar as well as turbulent flows. Most of the studies reported thus far were concerned with a 'smooth' interface, that is, the solid layer monotonously increases its thickness along the length of a pipe.

At a solid–liquid interface, the interactions among the flow, the shape of the interface and the heat transfer at the interface will be the dominant factor for a final steady-state profile. In reference to the mutual interactions, the studies of river ice [5, 6] indicated that the underside surface of ice cover, often, has a wavy or ripple undulation. Similar phenomena were observed in a laboratory on the ice surface on a cold plate in a water stream [7]. In those cases it has been noted that the undulation occurs only in a turbulent flow.

As for freezing in a pipe flow, Gilpin [8, 9] tested for the flow near or above transition Reynolds numbers and found that the steady-state ice profile produced a flow passage with a cyclical variation in cross-section along the length of the pipe, being called an ice-band structure; also, it was indicated that an extremely high value of friction factor was obtained due to a large-scale roughness caused by the undulation of the ice surface.

The main objective of the present study is to clarify the conditions for the onset of freeze-off of a pipe flow. The experiments were carried out to examine the ice profile, the heat transfer coefficient, and the pressure drop in a steady state. Analytical considerations based on the experimental results were also presented for the freeze-off conditions.

EXPERIMENTAL APPARATUS AND PROCEDURE

The experimental apparatus consists of a test section, a refrigeration unit and two circulation systems of water and coolant (30% $CaCl_2$ solution) as shown in Fig. 1. The circulation pipes of water and coolant, of which temperatures were controlled by PID-controlled heaters installed in each reservoir tank, were covered with insulation materials to decrease the heat flow from surroundings. The test section was constructed of two tubes in the vertical position; the inner one was a copper tube 16.6 mm (or 19.9 mm) I.D., 697 mm long and 1.1 mm wall thickness; the outer one was a 36 mm I.D. steel tube. The water in which freezing occurred was pumped through the copper tube and the coolant was circulated between the two tubes. The test section was installed with flange joints so that the ice in the pipe could easily be taken out for observation and measurement.

To minimize the heat flow through the flange to the test section, an acrylic resin plate was inserted between the flange couplings. A pressure tap was provided on this plate to measure a friction factor at the test section. The wall temperature of the copper pipe, T_w, was evaluated from three thermocouples located along the length of the pipe and the water temperature in the pipe, T_∞, was estimated from the mean value of the inlet and outlet of the test section. The total pressure at the inlet of the pipe, P_0, which was measured on the basis of outlet value, was controlled by the valve of the bypass pipe.

When the steady-state ice layer was obtained, the ice was taken out from the test section and the thickness

NOMENCLATURE

- B diameter ratio, d/D
- d minimum diameter at contraction region
- D diameter of pipe
- f friction factor of a pipe containing ice-bands
- h heat transfer coefficient at contraction region
- L length of pipe
- n number of ice-bands in the pipe, L/S
- Nu_d Nusselt number, hd/λ_w
- P_0 total pressure at inlet of a pipe measured on the basis of outlet value
- Re_d, Re_D Reynolds numbers, vd/ν, VD/ν
- Re_p Reynolds number based on P_0, equation (12)
- S spacing between ice-bands
- T_f, T_w, T_∞ freezing, pipe wall and water temperatures
- v mean velocity of water at contraction region
- V mean velocity of water in a pipe without ice
- x distance along pipe.

Greek symbols

- ρ density of water
- θ cooling temperature ratio, $(T_f - T_w)/(T_\infty - T_f)$
- λ_i, λ_w ice and water thermal conductivities
- λ^* thermal conductivity ratio, λ_i/λ_w
- ν kinematic viscosity of water.

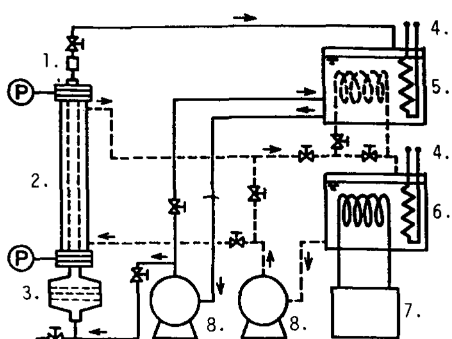

FIG. 1. Schematic representation of the experimental apparatus.

was measured with slide calipers. The experimental ranges covered for the two sets of test sections were:

$$D = 16.6 \text{ mm} \quad 4.8 \times 10^4 \text{ Pa} \leqslant P_0 \leqslant 2.5 \times 10^5 \text{ Pa}, \quad 1.5 \leqslant \theta \leqslant 27.8$$

$$D = 19.9 \text{ mm} \quad 8.5 \times 10^3 \text{ Pa} \leqslant P_0 \leqslant 2.5 \times 10^5 \text{ Pa}, \quad 3.9 \leqslant \theta \leqslant 19.2$$

where θ is a cooling temperature ratio defined by $\theta = (T_f - T_w)/(T_\infty - T_f)$.

As an aside the effect of free convection flow on the ice-layer profile was neglected in this experiment, since the temperature difference was small as $T_\infty - T_f = 0.4 \sim 1.0$°C and the Reynolds number range was $Re_d > 1.6 \times 10^3$.

RESULTS AND DISCUSSIONS

Pipe freezing process

The variation of flow rate in the pipe containing a growing ice layer is shown in Fig. 2. When the freeze-off does not occur, the flow rate shows a sinous variation during the first stage of ice growth, and finally approaches to a steady-state value. The cyclic increasing and decreasing of the flow rate gives rise to the appearance of the ice-band structure (see Figs 3 or 4). At the downstream of the contraction region, the heat transfer rate on the ice surface increases because of a separation flow due to a sudden enlargement of the flow passage. The ice in the separation flow region, therefore, melts away and as a consequence the ice-band migrates upstream. The variation of flow rate shown in Fig. 2 is caused by a change of friction factor in the ice-bands due to the appearance and migration. When the migration is stopped by establishing heat balance in the ice–water interface, the final steady-state ice layer is obtained; in the present experiment 7–16 h was needed to reach the steady state. On the other hand, when the freeze-off occurred, the flow rate decreased rapidly and the complete blockage of flow passage was observed in only 0.5–2 h.

The photographs in Fig. 3 show the profiles of steady-state and freeze-off ice layers, respectively. In

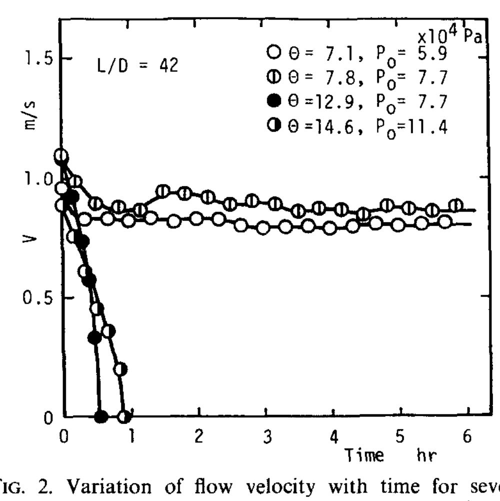

FIG. 2. Variation of flow velocity with time for several conditions of total pressure and temperature ratio.

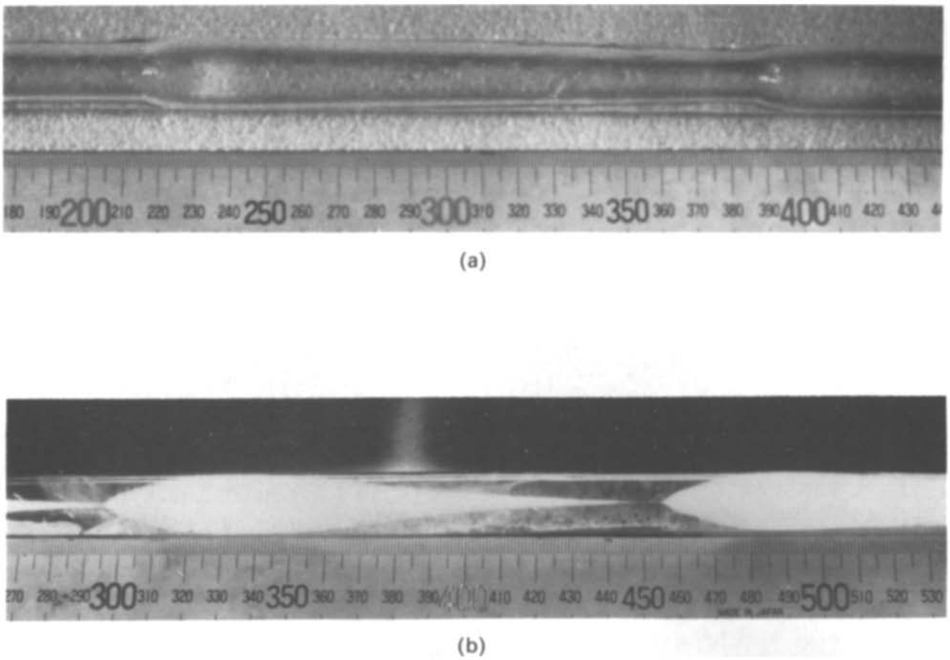

FIG. 3. Photographs showing typical ice profiles of (a) steady-state and (b) freeze-off conditions.

Fig. 3b it is suggested that the freeze-off occurs first at each contraction region in the ice-band and then develops to the freezing of pocket water trapped along the pipe. From these considerations the process to reach freeze-off is inferred as follows. The formation of ice-band results in a reacceleration of the flow in the contraction region [8]. Once the ice-layer thickness exceeds a certain value with further ice growth, the flow undergoes a rapid acceleration and relaminarization; then the heat transfer rate becomes smaller and the ice layer, thicker. Those changes result in larger friction factor, smaller flow rate and further growth of ice. The freeze-off, in consequence, occurs very quickly as shown in Fig. 2.

Steady-state ice layer

Typical profiles of the ice-band structure in a steady-state condition are shown in Fig. 4, in which the ice layer thickness is shown on an enlarged scale. It is indicated that the spacing of each ice-band becomes longer with decreasing Reynolds number, Re_D. For smaller Re_D the heat transfer rate in the separated flow region decreases, since the recirculating flow becomes weak. The migration of the ice-band in the upstream direction is, therefore, made weaker and longer ice-band spacing is obtained in the steady-state profile. In this case the ice surface downstream of the separation point formed a comparatively small expansion angle. Conversely, for larger Re_D the ice-band spacing becomes short and the

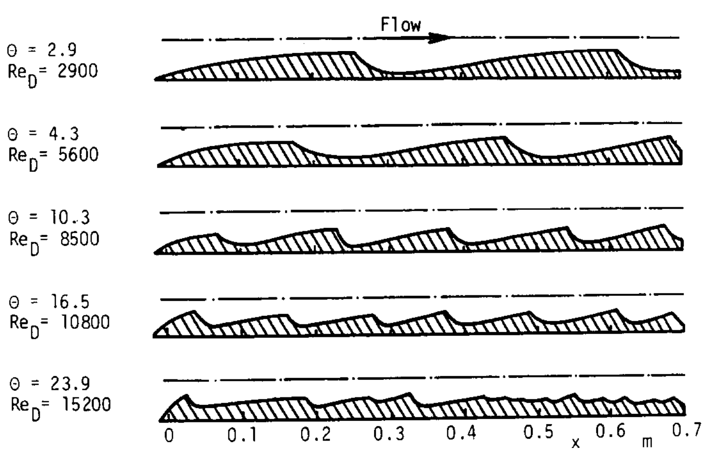

FIG. 4. Steady-state ice profiles for several conditions of Reynolds number and temperature ratio, $L/D = 42$ (ice thickness not in scale).

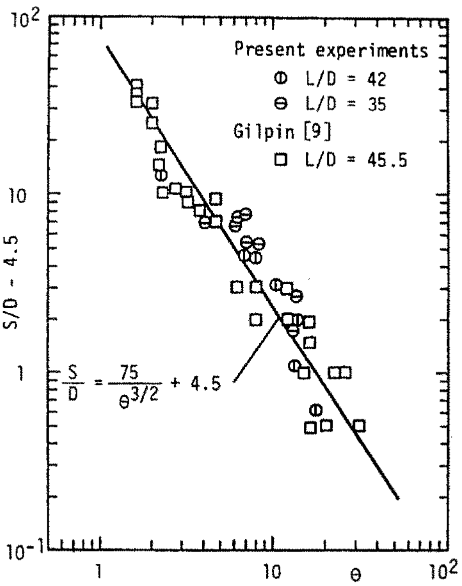

FIG. 5. The steady-state ice-band spacing plotted against the temperature ratio.

expansion angle large, because of larger heat transfer rate in the separated flow region. It can be seen from the result of $Re_D = 15{,}200$ that the regularity of the ice-band structure is broken in the region of $x > 0.2$ and that a three-dimensional profile with very small ripples is observed. The same phenomena have been observed in the ice surface on a cold plate in a water stream for, in particular, larger Reynolds number with strong turbulence [7].

In Fig. 5 the distance, S, between the separation points on the two-dimensional ice-bands is normalized by the pipe diameter, D, and is plotted as a function of cooling temperature ratio, θ. Gilpin [9] described that the ratio, S/D, appears to be largely independent of Re_D, since the Reynolds number loses importance in the highly accelerating and decelerating flow through a fully developed ice-band structure. It should be noted, however, that the range of Re_D is implicitly limited for which two-dimensional ice-band exists in the pipe. From the results in Fig. 5 the ice-band spacing is correlated as

$$\frac{S}{D} = \frac{75}{\theta^{3/2}} + 4.5 \quad \text{for} \quad 1.6 \leqslant \theta \leqslant 31. \tag{1}$$

The property of the ice-band structure of most interest concerning the freeze-off will be the diameter, d, of the narrowest water passage in the contraction region. At this region, the heat balance at the ice–water interface of a steady-state ice layer is described as follows: assuming that the heat flow in the axial direction in the ice is comparatively small for that of the radial one, we have

$$\pi d h (T_\infty - T_f) = 2\pi \lambda_i \frac{T_f - T_w}{\ln (D/d)} \tag{2}$$

where the LHS term indicates the convective heat flux transferred from the water to the ice surface and the RHS term, the conductive one through the ice. Let the Nusselt number be defined as $Nu_d = hd/\lambda_w$, equation (2) can be rewritten as:

$$Nu_d = -\frac{2\lambda^* \theta}{\ln B} \tag{3}$$

where λ^* is a thermal conductivity ratio of ice to water and B is a diameter ratio, d/D. The Nusselt number can, therefore, be calculated by measuring the temperature ratio and the minimum diameter of water passage in the ice-band. In Fig. 6 Nu_d is plotted against the Reynolds number defined by mean velocity, v, at the minimum diameter, in which the results for fully developed laminar and turbulent flows in a smooth pipe without ice are also shown. It should be noted that the present data show a smaller value than that of turbulent flow due to the laminarization of flow in the contraction region. A correlation for the Nusselt number was found to be

$$Nu_d = 0.0045 Re_d \quad \text{for} \quad 1.6 \times 10^3 \leqslant Re_d \leqslant 3.0 \times 10^4. \tag{4}$$

The pressure drop in a pipe containing cyclic two-dimensional ice-bands is caused mainly by two factors. One is due to viscous drag at the ice–water interface and the other is pressure loss occurring in the sudden expansion downstream of each separation point. The former is, in general, negligibly small compared to the latter and the pressure drop in the ice-band will be approximated by the factor of sudden expansion. In that case the pressure drop, Δp, caused by each ice-band will be

$$\Delta p = \frac{\rho v^2}{2} (1 - B^2)^2. \tag{5}$$

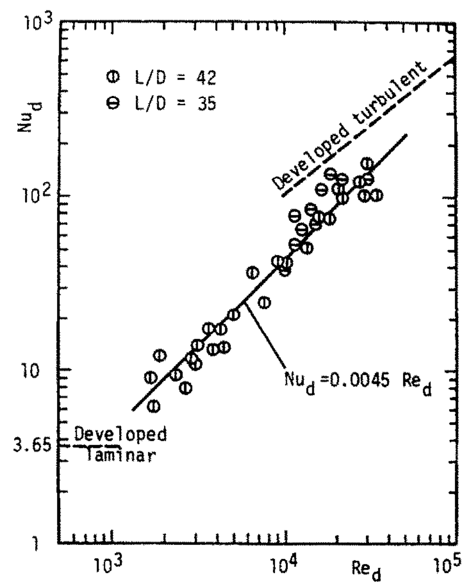

FIG. 6. The Nusselt number at the contraction region of the ice-bands plotted against Reynolds number.

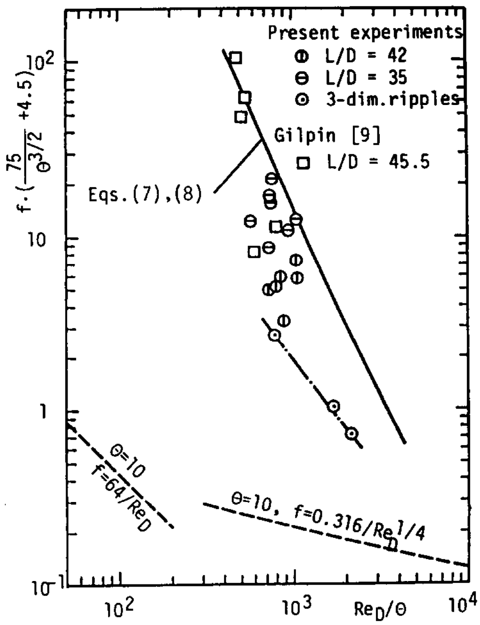

FIG. 7. Friction factor for a pipe containing a steady-state ice-band structure.

A friction factor, f, for a pipe containing n ice-bands is then

$$f = \frac{n\Delta p}{\dfrac{\rho V^2}{2}\dfrac{L}{D}} \tag{6}$$

where L is the pipe length. Introducing equations (1) and (5) into equation (6) and using the relation of $n = L/S$, we have

$$f\left(\frac{75}{\theta^{3/2}}+4.5\right) = \frac{(1-B^2)^2}{B^4}. \tag{7}$$

Making the substitution $Re_D = B\ Re_d$ into equations (3) and (4) yields

$$\frac{Re_D}{\theta} = -\frac{2\lambda^*}{0.0045}\frac{B}{\ln B}. \tag{8}$$

The friction factor can be expressed in terms of Re_D and θ by eliminating B from equations (7) and (8).

In Fig. 7 the predicted value of the friction factor is compared with the experimental data. It is seen that the predicted value overestimates the experimental data in the region of larger Re_D/θ. The value of Re_D/θ is approximately proportional to the diameter of flow passage, that is, inversely proportional to the ice thickness; therefore, for larger Re_D/θ the expansion angle becomes smaller than that of sudden enlargement and the pressure drop becomes smaller than the value estimated by equation (5). It can be said, however, that the predicted result gives an upper limit value of the friction factor.

The data of friction factor for three-dimensional ripples are shown in Fig. 7. In this case f becomes comparatively smaller because of the larger flow passage due to small ripples, as can be seen in Fig. 4. In any case, it should be noted that the pressure drop in a pipe containing ice-band structure is 10–100 times larger than that for a smooth pipe.

FREEZE-OFF CONDITIONS

For the case that the ice grows up to freeze-off, it is difficult to describe a hydrodynamic property because the flow velocity continuously changes to zero with increasing ice thickness. Consider that the ice-band is formed inside a pipe and that the ice is continuing to grow just before the freeze-off, the relation between the mean velocity, v, at the contraction region and the total pressure, P_0, at the inlet of a pipe can be written as

$$P_0 - n\Delta p = \frac{\rho v^2}{2} \tag{9}$$

where P_0 is the value measured on the basis of outlet total pressure. Substitution of equation (5) into equation (9) leads to

$$v = \frac{\sqrt{2P_0/\rho}}{\sqrt{1+n(1-B^2)^2}}. \tag{10}$$

The Reynolds number based on v can be expressed as

$$Re_d = \frac{B}{\sqrt{1+n(1-B^2)^2}} Re_p \tag{11}$$

where Re_p is called a modified Reynolds number and is defined by

$$Re_p = \frac{D}{\nu}\sqrt{\frac{2P_0}{\rho}}. \tag{12}$$

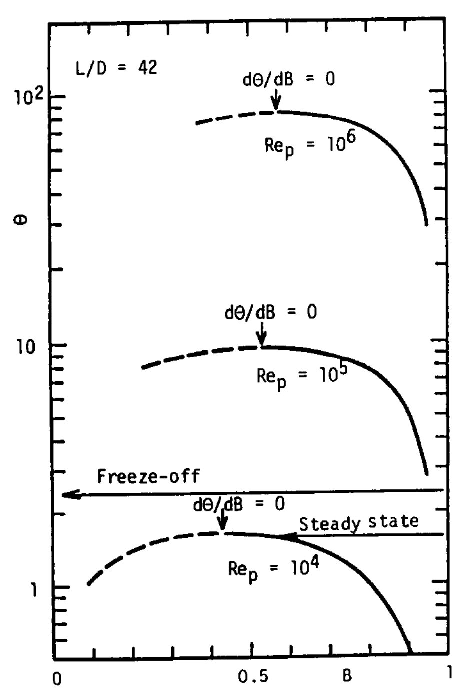

FIG. 8. Freezing process showing whether a freeze-off or a steady-state ice-band is reached.

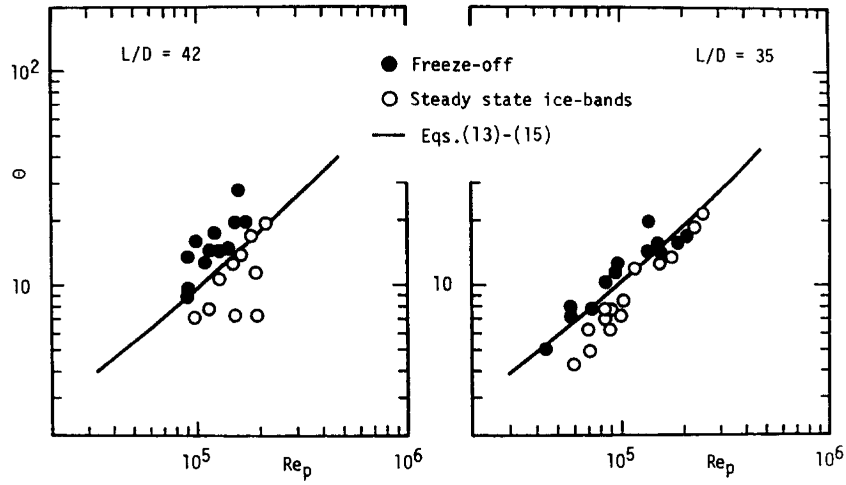

FIG. 9. A comparison between predicted and measured freeze-off conditions in a pipe.

For a pipe without ice ($B = 1$), it is realized from equation (11) that the value of Re_p agrees with that of Re_d.

It can be deduced that the freeze-off occurs when the thickness of ice ultimately exceeds the steady-state value expressed by equation (4). The relation for the steady-state condition is derived from equations (3), (4) and (11) as follows:

$$Re_p = -\frac{2\lambda^*\theta}{0.0045} \frac{\sqrt{1+n(1-B^2)^2}}{B \ln B}. \tag{13}$$

A relation between θ and the number of ice-bands, n, is given, by equation (1), as

$$\theta = \left[\frac{1}{75}\left(\frac{L/D}{n} - 4.5\right)\right]^{-2/3} \quad \text{for} \quad 1.6 \leqslant \theta \leqslant 31. \tag{14}$$

Figure 8 shows the relation of θ and B obtained by eliminating n from equations (13) and (14). For given conditions of Re_p and θ, the criterion whether the freeze-off occurs or not can be given by $d\theta/dB = 0$, as shown in Fig. 8. Differentiation of equation (13) with respect to B and introducing $d\theta/dB = 0$ leads to

$$n = -\frac{1+\ln B}{(1-B^2)^2+(1-B^4)\ln B}$$

$$\text{for} \quad B > 0.368 \quad \text{or} \quad n > 0. \tag{15}$$

The limiting value for which a steady-state ice-band exists can now be calculated from equations (13)–(15) by eliminating n and B. In Fig. 9, the predicted values for freeze-off are compared with the present experimental data. The solid and the open points indicate a freeze-off and a steady-state ice-band, respectively. The effect of L/D on freeze-off is apparently small because the factor of pipe length is implicitly included into θ which is estimated from the mean temperature of pipe inlet and the outlet. It can be seen from Fig. 9 that the predicted results for freeze-off agree well with the experimental data.

CONCLUSIONS

Experiments on the freezing of flowing water in a pipe was carried out at various temperature and flow conditions and the parameters concerned with pipe freeze-off were discussed using an analytical treatment of the steady-state ice-band structure. For the present experimental range, the following results can be drawn:

(1) The spacing of ice-band, S/D, is correlated as a function of cooling temperature ratio, θ, and the heat transfer coefficient at the contraction region of the ice-band is expressed by equation (4).

(2) The friction factor, f, for the pipe containing a steady-state ice-band is 10–100 times larger than that for a smooth pipe without ice, and the upper limit value of f is given by equations (7) and (8).

(3) The pipe freeze-off first occurs at the contraction region of the ice-bands; the freeze-off conditions in which a modified Reynolds number based on a total pressure is introduced, are represented by equations (13)–(15).

Acknowledgements—The authors gratefully acknowledge the support for this work by the research fund of the Ministry of Education, Japan (No. 58550147).

REFERENCES

1. R. D. Zerkle and J. E. Sunderland, The effect of liquid solidification in a tube upon laminar-flow heat transfer and pressure drop, *J. Heat Transfer* **90**, 183–190 (1968).
2. S. B. Thomason, J. C. Mulligan and J. Everhart, The effect of internal solidification on turbulent flow heat transfer and pressure drop in a horizontal tube, *J. Heat Transfer* **100**, 387–394 (1978).
3. M. N. Ozisik and J. C. Mulligan, Transient freezing of liquids in forced flow inside circular tubes, *J. Heat Transfer* **91**, 385–390 (1969).
4. Chul Cho and M. N. Ozisik, Transient freezing of liquids in turbulent flow inside tubes, *J. Heat Transfer* **101**, 465–468 (1979).
5. P. Rarsen, Hydraulic roughness of ice covers, *Proc. Am. Soc. civ. Engrs* **99**, 111–119 (1973).

6. G. D. Ashton and J. F. Kennedy, Ripples on underside of river ice covers, *Proc. Am. Soc. civ. Engrs* **98**, 1603–1624 (1972).
7. R. R. Gilpin, T. Hirata and K. C. Cheng, Wave formation and heat transfer at an ice–water interface in the presence of a turbulent flow, *J. Fluid Mech.* **99**, 619–640 (1980).
8. R. R. Gilpin, The morphology of ice structure in a pipe at or near transition Reynolds numbers, *A.I.Ch.E. Symp. Ser.* **75**, 89–94 (1979).
9. R. R. Gilpin, Ice formation in a pipe containing flows in the transition and turbulent regimes, *J. Heat Transfer* **103**, 363–368 (1981).

CONDITIONS DE GEL DANS UN TUBE TRAVERSE PAR UN ECOULEMENT D'EAU

Résumé—On étudie expérimentalement la structure à bandes du profil permanent longitudinal de la glace qui provoque une configuration cyclique de la section de passage de l'écoulement par succession de contractions et expansions de long du tube. On propose des équations pour l'espacement des bandes de glace, le coefficient de transfert thermique dans la région de contraction et le coefficient de frottement dans le tube. Les conditions de gel sont obtenues en introduisant un nombre de Reynolds modifié, basé sur une pression totale.

EINFRIERBEDINGUNGEN FÜR EIN WASSERDURCHSTRÖMTES ROHR

Zusammenfassung—Die Struktur der Eis-Schicht, durch welche das stationäre Eis-Profil eine Veränderung des Strömungsquerschnitts mit regelmäßigen Verengungen und Erweiterungen in Strömungsrichtung bewirkt, wurde experimentell untersucht. Für den Abstand der Eisschichten, für den Wärmeübergangskoeffizienten im Verengungsgebiet und für den Reibungsbeiwert im Rohr wurden Korrelationsgleichungen vorgeschlagen. Die Einfrierbedingungen wurden durch Einführen einer modifizierten Reynolds-Zahl, die auf dem Gesamtdruck basiert, ermittelt.

УСЛОВИЯ ВЫМЕРЗАНИЯ В ТРУБЕ, В КОТОРОЙ ТЕЧЕТ ПОТОК ВОДЫ

Аннотация—Экспериментально исследуется стационарная структура полос льда при циклическом чередовании сжатий и расширений по длине трубы. Предложены корреляционные уравнения для интервалов между полосами льда, коэффициента теплопереноса в зоне сжатия и коэффициента трения трубы. Введено модифицированное число Рейнольдса, основанное на суммарном давлении, и получены условия вымерзания в трубе.